TITLE: Series and Parallel Resistance

OBJECTIVES: (1) To provide hands-on experience in the interpretation of standard resistor nominal value and tolerance codes, and the use of the electrical experimenter’s breadboard to design and assemble DC resistive circuits;

(2) To provide hands-on experience in the use of an ohmmeter to measure the circuit characteristics of combinations of resistors in series, parallel, and series - parallel arrangements.

THEORY: Resistance; Resistors in Series and Parallel; Equivalent Series Circuits.

1. Theoretical Background. Ohm’s Law states that the resistance $R$ in a circuit or device is equal to the ratio of the potential difference $V$ across the circuit or device to the current $I$ flowing through the circuit or device. That is:

$$ R = \frac{V}{I} \quad \text{or, more commonly:} \quad V = IR $$

a. Series Resistance. When resistors are connected in series, the total current flowing in the circuit flows sequentially through each resistor in the circuit. The total potential difference (voltage drop) across the circuit is thus the sum of the potential differences (voltage drops) across each series resistor:

$$ \Sigma V = V_1 + V_2 + V_3 + ... = I(R_1 + R_2 + R_3 + ...) $$

The total resistance, $\Sigma R_s$, of resistors connected in series is given by:

$$ [1] \quad \frac{\Sigma V}{I} = \Sigma R_s = R_1 + R_2 + R_3 + ... + R_n $$

b. Parallel Resistance. When resistors are connected in parallel, the voltage drop across each parallel resistor is equal to the total voltage drop across the circuit. The total current flowing in the circuit is divided among the resistors in inverse proportion to the resistance of each. The total current is equal to the sum of the individual currents flowing through each parallel resistor:

$$ \Sigma I = I_1 + I_2 + I_3 + ... = \frac{V}{1/R_1 + 1/R_2 + 1/R_3 + ...} $$

The total resistance, $\Sigma R_p$, of resistors connected in parallel is given by:

$$ [2] \quad \frac{\Sigma I}{V} = 1/\Sigma R_p = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_n} $$
c. **Series - Parallel Resistance.** When resistors are combined in a circuit in a combination of series and parallel arrangements, the net circuit resistance generally is computed most easily by converting parallel resistances to equivalent series resistances, then computing the net equivalent series resistance of the circuit.

d. **Resistor Tolerance.** Resistors are manufactured to different standards of precision. The precision of a resistor is stated in terms of its tolerance as a percentage of its nominal resistance. Common tolerances (±) are 5%, 10%, and 20%. As with nominal resistance values, tolerance values are color-coded on the resistor body. **The net tolerance of combinations of resistors is computed using the procedures outlined in Appendix A to this laboratory.**

2. **Experimental Procedure.** In this laboratory we will: (1) determine the nominal values of resistance and tolerance of a number of resistors using their standard color coding, (2) measure their actual resistances using an ohmmeter, (3) compare nominal and measured values of resistance to determine whether each resistor's actual resistance is within the stated tolerance range, (4) assemble the resistors in series, parallel, and series-parallel arrangements on the experimenter's breadboard, and (5) measure the net circuit resistances across each of these circuit configurations to experimentally test the validity of the rules for combining resistors in series and parallel arrangements. The experimental values so obtained will then be compared to the computed theoretical values, and the resulting experimental error computed.

a. Determine the nominal values and tolerances of each resistor using the standard color codes. Record in the table below.

<table>
<thead>
<tr>
<th>Resistor</th>
<th>Nom. Resist.(Ω)</th>
<th>Tol.(%)</th>
<th>Act. Resist.(Ω)</th>
<th>% Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>1000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R2</td>
<td>470</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R3</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Measure the resistance of each resistor with the ohmmeter, and compute the % Deviation (% D) for each using the following. Record in the table below.

\[
\% D = 100 \left( \frac{\text{Actual Resistance} - \text{Nominal Resistance}}{\text{Nominal Resistance}} \right)
\]

(1) Set up a series circuit as shown on the final page.

(2) **Measure** the net resistance \( R_x \) across the entire circuit and record.
(3) Compute the theoretical net series resistance $\Sigma R_s = R$ of your series circuit using your measured values for each resistor, and compare to the resistance you measured across your entire circuit, $R_x$. Compute your experimental error using:

\[\text{[3]} \quad \%E = 100 \frac{(R_x - R)}{R}\]

(4) Compute the theoretical net tolerance ($\pm$) for your series circuit using the individual tolerances of each resistor. Compute the theoretical net series resistance of your circuit using the nominal values of resistance for each resistor. Compare this computed net nominal resistance of your circuit with its net measured resistance. Do the two values agree within the margin of net tolerance?

d. Parallel Resistance.

(1) Set up a parallel circuit as shown on the final page.

(2) Measure the net resistance $R_x$ across the entire circuit and record.

(3) Compute the theoretical net parallel resistance $\Sigma R_p = R$ of your parallel circuit using your measured values for each resistor, and compare to the resistance you measured across your entire circuit, $R_x$. Compute your experimental error using equation [3].

(4) Compute the theoretical net tolerance ($\pm$) for your parallel circuit using the individual tolerances of each resistor. Compute the theoretical net parallel resistance of your circuit using the nominal values of resistance for each resistor. Compare this computed net nominal resistance of your circuit with its net measured resistance. Do the two values agree within the margin of net tolerance?

e. Series - Parallel Resistance.

(1) Set up a series - parallel circuit as shown on the final page.

(2) Measure the net resistance $R_x$ across the entire circuit and record.

(3) Compute the theoretical net resistance $\Sigma R = R$ of this circuit using your measured values for each resistor, and compare to the resistance your measured across your entire circuit, $R_x$. Compute your experimental error using equation [3].

(4) Compute the theoretical net tolerance ($\pm$) for your series - parallel circuit
using the individual tolerances of each resistor. Compute the theoretical net series - parallel resistance of your circuit using the nominal values of resistance for each resistor. Compare this computed net nominal resistance of your circuit with its net measured resistance. Do the two values agree within the margin of net tolerance?

f. **Net Tolerance Computations.**

<table>
<thead>
<tr>
<th>Resistor</th>
<th>( R_{\text{nom}}(\Omega) )</th>
<th>Tol. (±%)</th>
<th>Max Deviation (±Ω)</th>
<th>( R_{\text{max}}(\Omega) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>1000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R2</td>
<td>470</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R3</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Series</th>
<th>Parallel</th>
<th>Series-Parallel</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Sigma R_{\text{nom}}(\Omega) )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Sigma R_{\text{max}}(\Omega) )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta R(\Omega) )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Sigma \text{Tol.}(±%) )</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

g. Summarize your results for this part and discuss your experimental results, including whether and to what degree your experiment results verified theory regarding combining resistors, and potential sources for experimental error in your experiment.

**Summary Table**

<table>
<thead>
<tr>
<th>Value</th>
<th>Series</th>
<th>Parallel</th>
<th>Series - Parallel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theor. Measured ( \Sigma R )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measured ( \Sigma R )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Error</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Theor. Measured ( \Sigma R )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Theor. Nominal ( \Sigma R )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Difference</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Sigma \text{Tol.}(±%) )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
h. In order to see how many possible values of resistance can be created using just three resistors, construct a table such as that below in which you list all possible combinations of one, two, and three resistors in series, parallel, and series-parallel circuit arrangements. Use parentheses to indicate parallel combinations – e.g. \((R1 + R2)\) would indicate the parallel combination of resistors \(R1\) and \(R2\). \(R1 + (R2 + R3)\) would indicate resistor \(R1\) in series with the parallel combination of \(R2\) and \(R3\). How many unique combinations are possible using just three resistors? Feel free to verify your results experimentally.

<table>
<thead>
<tr>
<th>Combination</th>
<th>Resistance ((\Omega))</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Individual</strong></td>
<td></td>
</tr>
<tr>
<td>(R1)</td>
<td></td>
</tr>
<tr>
<td>(R2)</td>
<td></td>
</tr>
<tr>
<td>(R3)</td>
<td></td>
</tr>
<tr>
<td><strong>Series</strong></td>
<td></td>
</tr>
<tr>
<td>(R1 + R2)</td>
<td></td>
</tr>
<tr>
<td>(R1 + R3)</td>
<td></td>
</tr>
<tr>
<td>Etc.</td>
<td></td>
</tr>
<tr>
<td><strong>Parallel</strong></td>
<td></td>
</tr>
<tr>
<td>((R1 + R2))</td>
<td></td>
</tr>
<tr>
<td>((R1 + R3))</td>
<td></td>
</tr>
<tr>
<td>Etc.</td>
<td></td>
</tr>
<tr>
<td><strong>Series-Parallel</strong></td>
<td></td>
</tr>
<tr>
<td>(R1 + (R2 + R3))</td>
<td></td>
</tr>
<tr>
<td>Etc.</td>
<td></td>
</tr>
</tbody>
</table>
Series and Parallel Resistance Laboratory

Series

Parallel

Series-Parallel
Appendix A: Computing the Net Tolerance of a Combination of Resistors

To compute the net tolerance of a combination of resistors:

a. Compute the net nominal resistance of the combination, $\Sigma R$.
b. Compute the maximum, net nominal resistance of the combination, $\Sigma R_{\text{max}}$, using the maximum tolerance specified for each resistor.
c. Compute the difference between these values, $\Delta R = \Sigma R_{\text{max}} - \Sigma R$.
d. Compute the net tolerance of the combination using:

$$\text{Net \% Tolerance} = \pm 100 \left( \frac{\Delta R}{\Sigma R} \right)$$

**Example:** Given two resistors with the values: 1200 $\Omega \pm 5\%$, 680 $\Omega \pm 10\%$:

(1) Net series tolerance:
- $\Sigma R = (1200 + 680) \Omega = 1880 \Omega$
- $\Sigma R_{\text{max}} = 0.05(1200 \Omega) + 1200\Omega + 0.10(680\Omega) + 680\Omega = (1260 \Omega + 748) \Omega = 2008 \Omega$
- $\Delta R = \Sigma R_{\text{max}} - \Sigma R = (2008 \Omega - 1880 \Omega) \Omega = 128 \Omega$
- Net % Series Tolerance $= \pm 100 \left( \frac{\Delta R}{\Sigma R} \right) = \pm 100 \left( \frac{128 \Omega}{1880 \Omega} \right) = \pm 6.81 \%$

(2) Net parallel tolerance:
- $\Sigma R = \left( \frac{1}{1200} + \frac{1}{680} \right)^{-1} \Omega = 434 \Omega$
- $\Sigma R_{\text{max}} = \left( \frac{1}{1260} + \frac{1}{748} \right)^{-1} \Omega = 469 \Omega$
- $\Delta R = \Sigma R_{\text{max}} - \Sigma R = (469 - 434) \Omega = 35 \Omega$
- Net % Parallel Tolerance $= \pm 100 \left( \frac{\Delta R}{\Sigma R} \right) = \pm 100 \left( \frac{35 \Omega}{434 \Omega} \right) = \pm 8.07 \%$