Experiment Report: Projectile Motion Experiment

Personnel: J. Boger

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Objective: Measure the range of a projectile for several different inclination angles and compare that to the range predicted by calculating it using measurements restricted to initial launch parameters.

Outline and procedure: The idea is to measure the initial launch parameters which include initial velocity $V_0$, angle of inclination and initial height of projectile, and use the projectile motion equations to solve for final range. This calculation has several measured parameters, each carrying some random error. I propagated these errors to an estimate of the final error in range. Then, to see how well I did, I measured the range directly. To get an estimate on the direct measurement of range, I fired the projectile 3 times and record the range using carbon paper on paper. The strike paper is attached to this paper for evidence.

The basic procedure was as follows.

1) Set the projectile launcher to the desired angle
2) test fire the projectile using the lowest spring setting, note where the projectile lands.
3) tape the strike paper at the location where the projectile will land, place carbon paper on top
4) Reload the launcher and cock device any remember to reset the photogate timer.
5) Fire the projectile, record time and landing mark on strike paper.
6) Repeat steps for 20, 30, and 40 degrees inclination.
Figure 1. Projectile range

Data table:

See Spreadsheet Projectile motion Table 1

Calculations

Starting with the equations for projectile motion:

\[ R_{\text{calc}} = x_0 + v_0 x_{\text{of}} \]
\[ y = y_0 + v_{0y} t_{\text{of}} - \frac{1}{2} gt_{\text{of}}^2 \]

From which we can solve the \( y \) equation for time of flight, \( t_{\text{of}} \).

\[ t_{\text{of}} = \frac{v_0 \pm \sqrt{v_0^2 + 2gy_0}}{g} \]
By vector components, the velocity is found in x and y directions.

\[ v_{0x} = v_0 \cos \theta \]
\[ v_{0y} = v_0 \sin \theta \]

The initial velocity is found from the photogate time and separation and calculated by,

\[ v_0 = \frac{L}{t_L} \]

Here \( t_L \) is the time it takes the projectile to go from one gate to another. With \( t_{of} \) calculated, we can calculate range.

\[ R_{calc} = v_{0x} t_{of} \]

**Error Calculations:**

I propagated the errors from measurements to \( R_{calc} \). Here is the principle error propagation formula for this.

\[
\Delta R_{calc} = R_{calc} \sqrt{\left( \frac{\Delta v_{0x}}{v_{0x}} \right)^2 + \left( \frac{\Delta t_{of}}{t_{of}} \right)^2}
\]

Where the error in \( t_{of} \) is:

\[
\left( \frac{\Delta t_{of}}{t_{of}} \right)^2 = \frac{1}{v_{0y}^2 + 2gy_0} \left( \frac{\Delta y}{t_{of}^2} + \frac{\Delta v_{0y}^2}{2gy_0} \right)
\]

and the error in \( v_{0x} \) is:

\[
\frac{\Delta v_{0x}}{v_{0x}} = \sqrt{\left( \frac{\Delta L}{L} \right)^2 + \left( \frac{\Delta \theta \sin \theta}{\cos \theta} \right)^2}
\]

This all comes from propagating errors in an algebraic formula. Unfortunately the errors are complicated due to a complicated formula used to process data.
Results:

Plot of Range as measured and as calculated. Errors overlap for 10 and 30 degree inclination data, but not for 20 and 40 degrees. I calculated accuracy of the data by assuming my measured value was truth. With that, the following accuracies were found.

<table>
<thead>
<tr>
<th>Accuracy at inclinations</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>3.96</td>
</tr>
<tr>
<td>20</td>
<td>13.2</td>
</tr>
<tr>
<td>30</td>
<td>1.54</td>
</tr>
<tr>
<td>40</td>
<td>3.54</td>
</tr>
</tbody>
</table>

Table 1. Accuracy in experiment.

The % accuracy matches the plot in that those value which overlap have a generally lower error.

Clearly the methods compare but I have made more error than is accounted for in my data. I suspect that the distances in truth have more error in them than I have allowed
for. Upon repeating the experiment, I would repeat the measurement of time between photogates and find an error in this time value. I noted during data collection that this value did vary and yet I considered it exact. In particular, I don’t believe my value for t in the 20 degree inclination data is correct. I adjusted this data value in the spreadsheet only to observe the effect and found that the plotted data point immediately jumped up to the expected value. This enforced my belief that the blunder is in this data point. (The original value for the tL data point is not altered in this report.) Repeating this one data point could close the 13% error seen in my data. Outside of this data point, the results are rather good showing the trend of Range with inclination. This lab was incredible, I wish I could do it everyday of the week for a month!

Additional sheets to this report:

1) 3 pages lab notes
2) Spreadsheet with calculations and data