1 Objectives

Upon successful completion on this laboratory you should be able to:

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1 Understand the principle of resonance
2 Understand the principle of impedance
3 Understand the principle of the Quality factor (Q-value) of a circuit.
4 Compute the resonant frequency of a series RLC circuit.
5 Compute circuit reactance, impedance and Q-value.
6 Use an ammeter and an oscilloscope to measure the frequency dependence of the current flowing in a series RLC circuit.
7 Gain familiarity with National Instruments Elvis II prototyping workstation.
8 Gain familiarity with National Instruments virtual instruments.

2 Background

2.1 AC Circuit Concepts

Two concepts that are fundamental to an understanding of resonance are reactance and phase difference, both of which we have studied in previous laboratories. We define the capacitive reactance in a circuit having capacitance \( C \) and signal frequency \( f \) as:

\[
X_C = \frac{1}{2\pi f C}
\]  \hspace{1cm} (1)

and inductive reactance in a circuit having inductance \( L \) and signal frequency \( f \) as:

\[
X_L = 2\pi f L
\]  \hspace{1cm} (2)

where reactance is measured in units of Ohms (\( \Omega \)). These components have a resistive effect in the circuit when AC signals pass through them.

From equations 1 and 2 we see that capacitive reactance is inversely proportional to the signal frequency, while inductive reactance is directly proportional to signal frequency. In other words, a capacitor blocks the flow on low or zero (DC) frequencies since it offers a large resistance at these frequencies. The inductor passes these frequencies as reactance is low when \( f \) is low. Conversely, the reverse is true when high frequency signals are applied.

In another preceding experiment we used the oscilloscope to measure phase differences in series RC and RL circuits. We may generalize the phase relationships in a series RLC circuit as follows:
• The instantaneous voltage across the resistance and the instantaneous current through the resistance are in phase with one another.

• The instantaneous current in the circuit has the same magnitude and direction in all parts of the circuit. Thus the phase of the current signal is the same in each component.

• The instantaneous voltage across the capacitor lags the instantaneous voltage across the resistance by 90 degrees.

• The instantaneous voltage across the inductor leads the instantaneous voltage across the resistance by 90 degrees.

• The voltage across the capacitance and the inductance are each 90 degrees out of phase with the voltage across the resistance, one lagging and one leading. Together they are 180 degrees out of phase with each other.

These relationships are conveniently displayed in a graphic format using phasors. A phasor displays a wave in vector form. The amplitude of the vector represents the amplitude of the wave and the angle the vector makes with the horizontal represents the phase of the wave. We have referred to these phasors in the impedance triangle.

Ohm’s Law, $V = IR$, also applies to reactive circuits as follows:

$$V_C = I_C X_C$$
$$V_L = I_L X_L$$

Where $V_C =$ voltage drop across the capacitor, $V_L =$ voltage drop across the inductor, $I_C = I_L = I_R$ current through the series components. These concepts are now applied to the concepts of reactance and phase differences to define resonance in a series RLC circuit.

### 2.2 Series Resonance

In a series circuit, voltages must sum to zero around any close loop in the circuit. This is Kirchhoff’s Voltage Law. Applying the loop equation to the RLC circuit gives:

$$v(t) = V_R \sin(\omega t + \phi_R) + V_L \sin(\omega t + \phi_L) - V_C \sin(\omega t + \phi_C)$$

This is a difficult equation to deal with so we use the concept of replacing the waves with vectors commonly referred to as phasors. This allows us to engage Ohm’s law as
given in equations 3 and 4. Doing this and incorporating the methods of adding vectors we get a simplified version of equation 5.

\[ V = \sqrt{V_R^2 + (V_L - V_C)^2} \]  

(6)

In this equation we can see that the source voltage is distributed across the three elements. However, if the voltage on the capacitance were equal to the voltage on the inductance, than all of the source voltage would be on the resistor. This is the resonance condition. To make that condition a reality, we only need for find a way to make \( V_C = V_L \). Getting a condition for these voltages to cancel each other is easily done by using Ohm’s law for the capacitance and inductance, equations 3 and 4 respectively.

\[ I_C X_C = I_L X_L \]  

(7)

Since the current must be the same in a series circuit, this leads to the condition for resonance, in which the capacitive reactance and the inductive reactance are equal.

\[ X_C = X_L \]  

(8)

Substituting equations 1 and 2 into equation 8 exposes the resonance frequency.

\[ f_0 = \frac{1}{2\pi\sqrt{LC}} \]  

(9)

In a series AC RLC circuit the voltages across the capacitance and the inductance, \( V_C \) and \( V_L \), are each 90° out of phase with the voltage on the resistance. But they lagging and leading \( V_R \) respectively so they are 180° out of phase with each other.

### 3 Procedure

Multisim can be used to do this experiment but it is good to get back to real 'hands-on' exercises so you should do this one with real components on a protoboard.

#### 3.1 Exercise 1: Voltage divider with a DC source

\[ R_1 = \ ]

\[ R_2 = \ ]

\[ R_L = \ ]

\[ L = \ ]

\[ C = \ ]
3.1.1 Questions for Exercise 1

1. What values did you choose for the resistors?

2. How much power was dissipated by resistor 1? \( P = VI \)

3. How much power was dissipated by resistor 2?

3.2 Exercise 2: RC circuit response to DC input

The RC circuit has a unique exponential time response due to the time it takes to charge the capacitor as seen in equations ?? and ?? . This exercise is intended to explore this phenomena through the simulation. Build the RC circuit shown in figure 4. Then measure the voltage response on the capacitor. When successful your scope trace should look similar to figure 2. To do this, adjust the timebase, frequency of input, R and C values until you have one full charge and one full discharge on the screen. This can be done by calculating the RC time constant \( (\tau = RC) \) and assuming that it will take 5 of those time periods \( (t = 5\tau) \) to fully charge, and 5 periods to fully discharge. The function generator should be on the square wave input.

\[ R = \text{______________} \]
1. Calculate the voltage on the capacitor after $t = 1\tau$.

2. There are measurement tools on the oscilloscope screen seen as red and blue vertical lines which can be moved by the mouse. Use these tools to measure the time constant. Start by placing the reference line on a zero value for voltage. Then use the relative measure and move out to the voltage calculated at $t = 1\tau$. Does the displayed $\Delta T$ match the calculated value?

3. Create and save a plot of your oscilloscope trace.

4. Now measure the voltage response for the resistor. To do this, it is helpful to add a DC bias to the square wave so that the reference wave is oscillating between 0 and $+10\,\text{V}$, rather than $\pm10\,\text{V}$. Create and save the plot of the oscilloscope trace for the resistor.

Figure 2: Oscilloscope trace for the RC circuit with a DC input

3.2.1 Questions for Exercise 2

1. What and why did you change the circuit to measure the voltage signal on the resistor?

2. When you measure the voltage on the trace after one time constant, did the time match the RC time constant?

3. Plot and explain the shape of the time response from the oscilloscope when you measured the voltage drop across the resistor.

4. How would you measure the current in the RC circuit using the oscilloscope?
3.3 Exercise 3: RC circuit response to AC input

An amazing thing happens when you switch the function generator feeding your RC circuit from the square wave to a sine wave. In short, the distorted response seen in figure 2 disappears and in its place is a distortion-free sine wave as in figure 3. When RC circuits are driven by sine waves they become voltage dividers in the AC world. While we will explore this phenomena more in later labs, we'll start looking at it today.

Change the input on the RC circuit such that an AC sine wave is the source. Measure the voltage signal across the capacitor on Channel B and put the initial wave from the function generator on Channel A. Measure the phase difference in the signals. Figure 3 shows this. Answer the questions for the capacitor signal, then change the circuit so as to measure the resistor signal and answer the same questions for the resistor.

3.3.1 Questions for Exercise 3

1. What is the difference in frequency between the source and the voltage signal across the capacitor?

2. What is the difference in amplitude between the source and the voltage signal across the capacitor?

3. What is the difference in phase between the source and the voltage signal across the capacitor?

4. What is the difference in frequency between the source and the voltage signal across the resistor?

5. What is the difference in amplitude between the source and the voltage signal across the resistor?

6. What is the difference in phase between the source and the voltage signal across the resistor?

7. How far apart are the capacitor and resistor signals in phase?

Figure 3: Oscilloscope trace for the RC circuit with an AC input

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Figure 4: RC circuits used in this lab.