Linear Algebra and Gaming

Bryan Lutgen

05/10/12
Abstract

This article will demonstrate how linear algebra is used with virtual environments in video games. It also has implementation of applied physics in a virtual environment as well as calculations for distance, speed, and direction.
1 Introduction

Video games are one of the most popular forms of entertainment today, but it wasn’t always like that. Before 1947, we did not have the technology to project objects on a screen that could be "interactive". Alan Turning, a British mathematician, developed a theoretical computer chess program as an example of machine intelligence. In 1947, Turing wrote the theory for a program to play chess. His colleague, Dietrich Prinz, wrote it as the first limited program of chess for Manchester University’s "Ferranti Mark 1". The program, however, was only capable of computing "mate-in-two" problems and was not powerful enough to play a full game. Technology has come a long way since then. The mathematics involved with gaming must be understood before a programmer can build a game. Gaming has two main aspects when it comes to design: Physical objects and the virtual environment. Because there is so much to gaming, we will focus on the virtual environment rather than the creation of the objects.

2 Vectors

When someone plays a game, what do they see? Usually, the first thing they see is all the physical attributes of the game. Objects are created using vectors, but we will go over what you cannot see. All video games rely on vectors. When someone plays a game, there are different options that the game gives them. They can play in the first or third person view, or they might have the option to pick up a weapon with certain attributes. Vectors determine what a character looks like, the position the character is at, the direction its facing, and the characters speed. For programmers, its important to understand Linear Algebra and the role it plays in game creation.

The picture shown below is showing three different vectors used in different ways. The position vector indicates that the man is standing four units east of the origin and two units north. The velocity picture shows that in one unit of time, the car moves three kilometers north, and two units west. The direction vector tells us that the Dragon is pointing east. Because vectors are used in different ways, we need to know how to manipulate them.
3 Vector Addition/Subtraction and Scalar Multiplication

Vector addition and vector multiplication is very useful in game creation. For starters, one needs to know how to add vectors together. Each component is added separately in the formula. For example:

\[
\begin{pmatrix} 1 & 3 & 2 \\ \end{pmatrix} + \begin{pmatrix} 1 & -2 & 4 \\ \end{pmatrix} = \begin{pmatrix} (1 + 1) & (3 - 2) & (2 + 4) \\ \end{pmatrix} = \begin{pmatrix} 2 & 1 & 6 \\ \end{pmatrix}
\]

One of the most common applications in games for vector addition is physics. Any physically based object will have vectors for position, velocity, and acceleration. For every frame, we have to add the velocity to the position, and the acceleration to the velocity. Consider the example of a character jumping. As the character starts the jump, his velocity is calculated by moving one space to the right and three spaces vertically. This can also be put into terms of a vector, \((1 \ 3)\). Because of a gravitational effect, we want to incorporate a vector representing gravity, \((0 \ -1)\). In each frame, it shows the character’s new velocity vector for each segment of time. Remember, these are not position vectors, but velocity vectors.
Multiplication works in almost the same way. Taking from the previous example, say the character jumps when he is over a special area, like a spring. In this case, we can multiply the distance he jumps by a certain vertical amount. Let’s say that the spring multiplies the character’s jump by 3 units vertically. This would make the initial jump start at velocity (1 9) instead of (1 3). After the initial jump, the gravitational forces vector would take into effect:

\[
(1 \ 9) + (0 \ -1) = ((1 + 0) \ (9 + -1)) = (1 \ 8)
\]

It’s at this point that we would continue until the character touches the virtual ground.

Subtraction works in the same way as addition: subtracting one component at a time. Vector subtraction is useful for getting a vector that will point from one position to another. For example, let’s say a character (Blue Guy) is standing at (3 2) with a bow, and an enemy (Red Guy) is at (4 3). To determine the vector that the arrow must travel to hit the red guy, we can subtract the blue guy’s position from the red guy’s position. This gives us:

\[
(4 \ 3) - (3 \ 2) = ((4 - 3) \ (3 - 2)) = (1 \ 1)
\]
4 Distances

Instead of calculating vectors, we also need to know how to calculate distances. There is a different formula to use called the magnitude. The magnitude is a type of scalar in the form:

\[ ||V|| = ||(a \ b)|| = \sqrt{(a \ b) \cdot \begin{pmatrix} a \\ b \end{pmatrix}} = \sqrt{a^2 + b^2} \]

Example:

If a bomb goes off with a radius of 2 units, will it damage the character?

Given:
- Character positioned at (3 3): Vector C
- Bomb positioned at (1 2): Vector B
To find this, we subtract $C$ and $B$ to get the vector between them. After finding the vector, we find the length using the formula for magnitude. This is the formula used for the character and the bomb:

$$||C - B|| = ||(\begin{pmatrix} 3 \\ 3 \end{pmatrix}) - (\begin{pmatrix} 1 \\ 2 \end{pmatrix})|| = ||(\begin{pmatrix} 2 \\ 1 \end{pmatrix})|| = \sqrt{2^2 + 1^2} = \sqrt{5} = 2.23$$

If the bomb has an explosion radius of 2 units, the character would not be harmed as he is far enough away.

5 Conclusion

In a game, vectors are not only used for creating objects to look at, but they are used for physical characteristics such as simulated movement, gravity, velocity, acceleration, and projection. We have only covered the basic topics of how linear algebra is used with video games, but technology has come a long way over the past five decades. Without mathematicians like Alan Turning, we would still be looking at motion pictures through a projection screen. Linear algebra is the key to game creation, and without the knowledge behind it, one can’t fully appreciate a game’s design. Not just the physical traits of the game, but the unseen characteristics. Hopefully, with the knowledge gained from reading this article, one can understand the basic mathematics behind the creation of a virtual environment.
6 Works Cited